Calculus 2 - Test 2 Review Key

Dr. Graham-Squire, Spring 2020

1. Evaluate the integrals.

   (a) $\displaystyle\int_1^{\sqrt{2}} \frac{x^5}{\sqrt{4-x^2}}\,dx$ (Round to nearest .001)
   Ans: Use trig substitution (easier) or integration by parts (harder). Final answer is 0.758.

   (b) $\displaystyle\int_0^2 \frac{3}{\sqrt{2-x}}\,dx$
   Ans: This is an improper fraction because the function is not continuous at 2. Need to use limit notation to get full credit, final answer is $6\sqrt{2}$

2. (a) Use the Midpoint Rule with six subintervals (M_6) to approximate $\displaystyle\int_0^3 \frac{dt}{1+t^2+t^4}$.

   Ans: $0.89548 = M_6$

   (b) Use the error estimate for the midpoint rule to estimate how much error you might have in your answer from (a). You will need to use Maple for that.

   Ans: The error formula is $|E_M| = K(b-a)^3/(24n^2)$. $b-a=3$ and $n=6$. To find K we use Maple to find the second derivative of $\frac{1}{1+t^2+t^4}$, which is...ugly, and I don't want to type it all in. In any case, I graph it from $x=0$ to $x=3$ and the graph has a low of -2 at $x=0$ and a high of 10/9 at $x=1$. I need K to be greater than or equal to the absolute value of both of those, so I choose $K=2$. My answer is then

   $$E_M \leq \frac{2(3^3)}{24(6^2)} = 0.0625$$

   (c) Use Maple to calculate the actual integral. How far off from the correct value is your answer from (a)? Is it less than the error from (b)?

   Ans: actual numerical integral is 0.89537, which is well within 0.0625 from the estimate (it is actually less than 0.0001 away).

3. Sketch the region enclosed by $y = x^3 - 9x$ and $y = -5x$ and then find its area. (Note: the answer is <u>not</u> zero).

   Ans: Need to calculate $\displaystyle\int_{-2}^{0} [(x^3-9x)-(-5x)]dx + \int_0^2 [(-5x)-(x^3-9x)]dx$. Final answer is 8.

4. Consider the region W bounded by $y = \frac{1}{x}$, $y=0$, $x=1$ and $x=3$. Find the volume of the solid obtained by rotating W about (a) the line $y=-3$ and (b) the y-axis.

   Ans: (a) Using washers, get $\displaystyle\int_1^3 \pi[(3+\frac{1}{x})^2 - 3^2]\,dx = \pi(6\ln 3 + (2/3))$.

   (b) Using shells, get $\displaystyle\int_1^3 2\pi x(\frac{1}{x})\,dx = 4\pi$.

   Can also use washers to get $\displaystyle\pi\int_0^3 (3^2-1^2)\,dy + \pi\int_{1/3}^1 (\frac{1}{y^2}-1)\,dy = 4\pi$